DETERMINATION OF STRESSES BEHIND A SHOCK WAVE FRONT IN SNOW POWDER AVALANCHE WITH ALLOWANCE FOR THE FIRMNESS OF THE SOLID PHASE

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A two-phase continuous medium composed of solid particles uniformly distributed in air, such as a snow avalanche, is examined.

Problems associated with the determination of the stresses behind a shock front in such media are encountered in many cases, in particular in the interaction between a snow avalanche and a solid obstacle.

It will be shown in the following that for sufficiently low densities of the mixture, the total stress behind the shock front is almost independent of the stresses which arise in the interaction between solid particles, even for strong condensations of the medium behind the shock front.

We assume that the gas in the space between the solid-state particles is an ideal gas, that the relative velocities of the solid and gas phases are negligible everywhere, except in the zone near the shock front, and that the volume compressibility of the solid phase is negligible compared with the compressibility of air.

The relations at the shock wave with allowance for the elastic interaction between the solid particles have the form

$$\rho_0 (V_0 + D) = \rho_1 D, \quad p_1 = \frac{mRT\rho_1}{1 + m - \rho_1/\rho^\circ}$$

$$\sigma_0 + p_0 + \rho_0 (V_0 + D)^2 = \sigma_1 + p_1 + \rho_1 D^2 \tag{1}$$

If the function $\sigma = \sigma(\rho, T)$ is known, these relations are closed. From (1) it follows that

$$p^* = p_0 + \frac{V_0^2 \rho_0 \rho_1}{\rho_1 - \rho_0}, \qquad p^* = p_1 + \sigma_1 - \sigma_0 \tag{2}$$

Obviously, there exists a certain density $\rho^*$ (for snow, $\rho^* \simeq 0.2$), such that for densities of the mixture smaller than $\rho^*$, the medium under consideration does not experience static loads. A qualitative plot of the relation $\sigma = \sigma(\rho)$ is shown in Fig. 1.

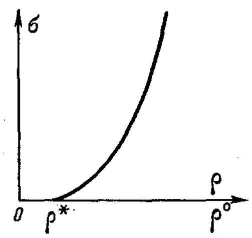


Fig. 1

The following analysis will be limited to densities smaller than $\rho^*$. For such densities, $\sigma_0 = \sigma_0(\rho_0) = 0$.

Let us examine the function

$$p^* = p_1 + \sigma_1 \tag{3}$$

A qualitative plot of $p_1 = p_1(\rho)$ is shown in Fig. 2 by the solid line. The curve $p^* = p^*(\rho)$ is shown in Fig. 2 by the dashed line. It is obvious that for densities below $\rho^*$, the curves (1) and (2) overlap.

Let us examine the right-hand side of (2) for densities $\rho_1$ behind the shock wave greater than the initial density

$\rho_0$. Its curve is shown in Fig. 3. The curve has a vertical asymptote $\rho_1/\rho_0 = 1$ and a horizontal asymptote $p_* = p_0 + \rho_0 V_0^2$.

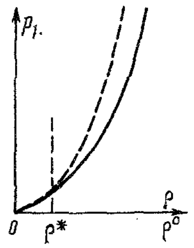

Fig. 2

The value of p* which defines the total stress behind the shock wave corresponds to the point of intersection of the curve in Fig. 3 with the dashed curve in Fig. 2.

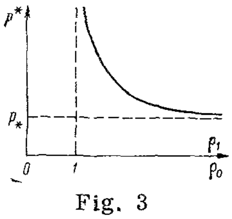

Fig. 3

Curves (1) and (2) can intersect in two cases (Fig. 4):

1) $\rho_1/\rho_0 < \rho^*/\rho_0$ (intersection of the curves (1') and (2) in Fig. 4),

2) $\rho_1/\rho_0 > \rho^*/\rho_0$ (intersection of the curves (1") and (2) in Fig. 4).

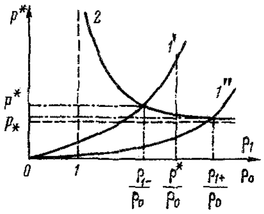

Fig. 4

In the first case, allowance for firmness is not essential, since $\rho_1 < \rho^*$ and $\sigma = \sigma(\rho_1) = 0$. In the second case, the value of p that corresponds to the point of intersection of curves (1") and (2) (Fig. 4) differs only slightly from the horizontal asymptote $p_* \simeq p_0 + \rho_0 V_0^2$, since by definition the initial density of the medium is small, i.e., $\rho^*/\rho_0 \gg 1$, and curve (2) (Fig. 4) is already close to its horizontal asymptote.

Consequently, in the second case, the value of p* is altogether independent of the shape of curve (1") (Fig. 4). For example, the curve $p_1 = p_1(\rho_1)$ can be taken as this curve. Only the density $\rho_1$ of the medium behind the shock wave depends on the form of function $p^*(\rho)$.

Thus, for sufficiently small initial densities of the mixture, the total stress at the obstacle can be determined with satisfactory accuracy, regardless of the form of function $\sigma$. Specifically, one may set $\sigma \equiv 0$. The error does not exceed 20%, even for initial density of the mixture $\rho_0 \simeq 0.1$ g/cm$^3$, $\rho^* = 0.2$ g/cm$^3$, $p_0 = 1$ technical atmosphere, and a velocity $V_0 \leq 50$ m/sec.

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